Sayılar Teorisi Dersi Final Cevap Anahtarı

1- $4x+3y-6z=8$

$a_1=4,\ a_2=3,\ a_3=-6 \qquad (4,3,-6)=d=1$

$(3,-6)=d_1=3,\ \beta=-\frac{a_3}{d_1}=2 \qquad \delta=1$

$\alpha-2\gamma=1,\ \alpha=3,\ \gamma=1$

$y=\alpha t+\beta u=3t+2u$

$z=\gamma t+\delta u=t+u$

$4x+3(3t+2u)-6(t+u)=8$

$4x+3t=8 \qquad x_0=5 \quad t_0=-4$

$x=5+3v \qquad t=-4-4v$

$x=5+3v$ // $\quad y=-12-12v+2u$ // $\quad z=-4-4v+u$ //

2- a) $2^{18}\equiv 1 \pmod{19}$ Fermat Teo.

$2^{17}+6\equiv x \pmod{18} \qquad \varphi(9)=6$

$x\equiv(2^6)^2\cdot 2^5+6\equiv 2 \pmod 9 \qquad x=2,11 \qquad x$ çift

$x\equiv 2 \pmod{18}$

$2^{2^{17}+6}+1\equiv 2^2+1\equiv 5 \pmod{19}$ bulunur.

b) $[3,5,2,1,4]=?$

$$3+\cfrac{1}{5+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{4}}}}=\frac{239}{75}$$

3- a) $-\dfrac{86}{31} = -3 + \dfrac{7}{31} = -3 + \dfrac{1}{\frac{31}{7}} = -3 + \dfrac{1}{4+\frac{3}{7}}$

$= -3 + \dfrac{1}{4+\frac{1}{\frac{7}{3}}} = -3 + \dfrac{1}{4+\frac{1}{2+\frac{1}{3}}} = [-3, 4, 2, 3]$

b) $\sigma(p^2q^2) = \dfrac{p^3-1}{p-1} \cdot \dfrac{q^3-1}{q-1} = (p^2+p+1)(q^2+q+1) = 7.31$

$p^2+p+1=7$, $q^2+q+1=31$ $p=2$ $q=5$ bulunur.

4- $6x^3+27x^2+17x+20 \equiv 0 \pmod{30}$

$30 = 2.3.5$

$6x^3+27x^2+17x+20 \equiv 0 \pmod{2}$ $b_1 = \bar{0}, \bar{1}$

$6x^3+27x^2+17x+20 \equiv 0 \pmod{3}$ $b_2 = 2$

$6x^3+27x^2+17x+20 \equiv 0 \pmod{5}$ $b_3 = \bar{0}, \bar{1}, \bar{2}$

olup 6 çözüm var bir tanesini yapalım

$x \equiv 1 \pmod{2}$ $M_1 = 15$ $M_2 = 10$ $M_3 = 6$

$x \equiv 2 \pmod{3}$ $a_1 = 1$ $a_2 = 2$ $a_3 = 2$

$x \equiv 2 \pmod{5}$ $c_1 = 1$ $c_2 = 1$ $c_3 = 1$

$15c_1 \equiv 1 \pmod{2}$ $\bar{x} = 15 + 20 + 12 = 47 \equiv 17 \pmod{30}$

$c_1 = 1$ Diğer çözümler

$10c_2 \equiv 1 \pmod{3}$ $x \equiv 2, 5, 11, 20, 26 \pmod{30}$

$c_2 = 1$ Bulunur.

$6c_3 \equiv 1 \pmod{5}$